Introduction

Elastic plastic fracture mechanics (EPFM) is a domain of fracture mechanics, which considers extensive plastic deformation ahead of a crack-tip prior to fracture. It is well known that J-integral (J) and crack-tip opening displacement, CTOD (δ) can be used as fracture parameters for analysis of fracture problems under EPFM. In EPFM it is required that J and δ should be interchangeable to each other[1]. Thus, it is essential to examine the relationship between J and δ. A well-known general relationship between J and δ given by T.L. Anderson[2] is:

\[ J = m \sigma_y \delta \] ..................................(1)

where, \( \sigma_y \) is the yield stress of the material, \( m \) is constant and \( \delta \) is CTOD. Earlier literature[3-5] indicates that the load intensity measured in terms of J-integral as a single parameter alone does not describe stress/strain field ahead of the crack-tip uniquely and accurately. Therefore, there is a necessity of introducing a second parameter with J, which is required to characterize the crack-tip fields. Interestingly, the constant factor \( m \) in the relationship between J and δ given in Eqn. (1) is known to be constraint dependent. Thus \( m \) can serve as a parameter to characterize constraint constraints[5]. Shih[6] has shown that the relationship between J and δ can be obtained theoretically by HRR stress field equations[7-8] as:

\[ \delta = d_e \frac{J}{\sigma_y} \] ..................................(2)

where, \( d_e \) is a constant, which depends on Ramberg-Osgood (R-O) constant \( n \) of the material. From Eqn. (1) and Eqn. (2) the relation between \( m \) and \( d_e \) is:

\[ m = \frac{1}{d_e} \] ..................................(3)

Shih[6] has also shown that \( d_e \) usually varies between 0.4 to 0.8 for common structural steels and for fully plastic materials \( (n=\infty) \) \( d_e=1 \), which is obtained by extrapolation. Omidvar et al.[9] using closed form solutions have confirmed that their results on relationship between J and δ fully corroborate the results of Shih[6]. The analytical solutions and 2D finite element analysis indicate, the magnitude of \( d_e \) is found to be dependent on the strain hardening component (n) of the material and specimen a/W ratio. Shih[6], Omidvar et al.[9] Panontin et al.[10] and Kulikarni et al.[11] have investigated the validity of J as a fracture parameter and the J-δ relationship for the determination of critical CTOD (δ) in predominantly plane stress fracture. As \( d_e \) can be used as a constraint parameter, it is required to examine the effect of state of stress and different material on the factor \( d_e \), which can address constraint effects. Chiodo and Ruggieri[12] have argued that a full set of J and CTOD solutions for varying crack geometries and loading modes directly connected to the description of fracture behaviour under large scale yielding condition is still lacking. Enyang Wang et al.[13] have shown that marked variability of the predictive accuracy of Eqn. (1) implies that this equation, which was developed based on 2D plane-strain FEA, is not adequate to predict the CTOD value for 3D Single Edge Tensile (SE(T)) specimens. Work is currently being carried out to develop a more
accurate equation to predict CTOD from \( J \) based on the 3D Finite Element Analysis (FEA) results. Yifan Huang and Wenxing Zhou,[14] have studied finite element analyses with the large-displacement/large-strain formulation performed on clamped SE(T) specimens to evaluate the plastic constraint \( m \) factor that relates CTOD to \( J \). The analysis results suggested that the value of \( m \) depends on strain hardening component \( n \) and specimen configuration (i.e. \( a/W \), \( B/W \) and side-grooving). As per the best knowledge of authors there is a need to examine the relationship between the \( J \) and \( \delta \) for varied \( a/W \) and \( \sigma \), on magnitude of \( d_e \) for fracture toughness analysis of different materials using EPFM. The major emphasis in this investigation is on the study of constraints for two different materials (HY80 and A302 steels) and comparison with IF steel through different state of stress using ABAQUS software.

The objective of this investigation is to compute \( J \) and CTOD for various applied loads using 2D elastic-plastic finite element analyses considering CT specimen geometry of various \( a/W \) and different state of stress for two different materials. To examine the relationship between the \( J \) and \( \delta \) for varied \( a/W \) and \( \sigma \), on magnitude of \( d_e \) computed by plastic hinge model for different materials.

**Finite Element Analysis**

The finite element computations were performed using ABAQUS software.[15] A series of 2D elastic plastic FE analyses have been conducted on the CT specimens subjected to various applied load steps and details of the analyses are discussed in this section.

**Material and Specimen Geometry**

A302 steel and HY80 steel have been considered for the FE analyses. The values of mechanical properties of HY80 and A302 are obtained using pixel method from the work of Joyce and Link[16] and O’Dowd[17].

The geometry of the CT specimen used in this FE analyses is shown in Figure 1 and is in accordance with ASTM standard E1820-13[18]. The width of the specimen \( W=20\text{mm} \) is used for the analysis and other dimensions are computed accordingly.

The plastic deformation between two successive points in the model was assumed to be linear with a particular tangent modulus. In the Elastic-plastic analysis the Elastic properties as well as the true stress strain values for both HY80 and A302 steel were fed in to the models, developed in the initial phase. The applied stress \( (\sigma) \) in the analysis is computed with the analytical formulation provided in the work of Priest[19]. Different load steps were imposed during each of the finite element analysis in a manner that the magnitude of normalized applied stress \( (\sigma/\sigma_t) \) remains in the range 0-0.80. Because of mesh quality, plasticity and constraints the maximum load being limited to \( \sigma/\sigma_t \) less than or equal to 0.80. For each load step \( J \) and Crack Mouth Opening Displacement (CMOD) values were extracted using the postprocessor of ABAQUS software.

**Mesh and Boundary Conditions**

Finite element computations were carried out considering only one-half of the specimen geometry due to symmetry. The 2D analysis domain is decritized using 8-noded quadrilateral finite elements using reduced integration (element type CPE8R for plane strain and CPS8R for plane stress within the ABAQUS library) are used. The use of these kinds of elements were found in Kim et al.[20]. In this study, very fine mesh was used in the region around the crack tip to achieve better results. Courtin et al.[21] have compared various methods (Empirical expressions, 2D singular elements, Displacement extrapolation and J-integral) to extract stress intensity factor both in 2D and 3D crack configurations and reported that it does not require an excessive mesh refinement since the results obtained with the coarse mesh are in good agreement with fine meshing while using ABAQUS software. Typical FE mesh generated by taking origin at the crack-tip along with boundary condition is clearly depicted in Figure 2.

Due to half symmetry, the symmetrical displacement boundary conditions have been imposed \((u_y=0)\) along the ligament of the model. The load applied through pin holes is simulated by applying point loads on the circumference of the pin hole, which approximately subtends to \( 60^\circ \) to keep the loading in mode-I. The nodes of loading pins holes are given displacement boundary conditions having \( u_x=u_y=0 \) and \( u_{z} \neq 0 \). A series of elastic-plastic stress analyses on compact tensile (CT) specimen (Figure 1.) of thickness 3 mm (plane stress and plane strain with thickness) and \( a/W=0.45 \) to 0.65 in steps of 0.05 are carried out for different applied load levels. In these analyses for every load step, elastic plastic fracture parameters J-integral and CMOD are extracted.

**Extraction of \( J \) values**

In this study, plane-stress and plane-strain elastic plastic FE analyses have been conducted on the CT specimen with \( a/W=0.45 \) to 0.65 to extract the \( J \) and CTOD values for A302 and HY80 steel. In the present 2D analysis, using domain integral method the ABAQUS software automatically finds user defined five contours in order to carry out \( J \)-integrals. It is widely accepted that the first few contour do not provide consistent results because of numerical singularities [21]. Therefore, the magnitudes of first two contours have been neglected in the analyses, in order to get a convergent value of \( J \). The mean value of rest of three contours is computed. ABAQUS provides a procedure for numerical evaluation of the \( J \), based on the virtual crack extension/domain integral methods [22-23]. The method is particularly attractive because it is simple to use and provides excellent accuracy, even with rather coarse mesh.
Crack-tip opening displacement CTOD ($\delta$)

The magnitude of CTOD for various load steps have been estimated by conversion of crack mouth opening displacement (CMOD) to CTOD using rotation factor, which is popularly referred as plastic hinge model and used in experimental fracture analysis [24].

\[
\delta = \frac{(CMOD) a}{r b} \]

(4)

where $r$ is rotation factor, the value of $r$ according ASTM E1290 varies with specimen $a/W$ ratio and is between 0.44-0.47 for CT specimen, $b$ is the un-cracked ligament and $a$ is the crack length of the specimen. The Eqn. (4) estimates only the plastic part of CTOD, as the investigation is elastic-plastic analysis; the elastic part of CTOD is found to be insignificant and is neglected in the present work. The value of $\alpha$ and $r$ can be obtained by from Eqn. (5) and (6):

\[
\alpha = 2 \left( \frac{a}{b} \right)^2 + \left( \frac{a}{b} \right) + \left( \frac{1}{2} \right) - 2 \left( \frac{a}{b} \right) + \left( \frac{1}{2} \right)\]

(5)

\[
r = 0.4(1 + \alpha)\]

(6)

Results and Discussion

The variations of $J$ values of the specimen with normalized applied load, $\sigma/\sigma_y \leq 0.80$, and $a/W=0.45$ to 0.65 in steps of 0.05 were studied for plane stress and plane strain for A302 and HY80 Steel material. A typical variation of $J$ vs. $\sigma/\sigma_y$ for various $a/W$ ratio, which indicates the LEFM conditions ahead of crack tip. For $\sigma/\sigma_y > 0.07$, it is observed that the variation of $J$ is nonlinear with applied load demonstrating the regime of EPFM. It is interesting to note from figures that for the same applied load the intensity of $J$-integral depends on the $a/W$ ratio of the specimen, material and state of stress. For the similar applied loads $J$ is large for the specimen with large $a/W$ ratio. In order to understand the variation of $J$ vs. applied load for different materials, specimens with a particular $a/W$ are studied. A typical variation for $a/W=0.50$ is shown in Figure 6.

![Image](https://example.com/image1.png)

**Figure 3:** A typical deformed specimen compared to un-deformed specimen.

In the plastic hinge model CMOD is converted to CTOD using rotation factor. At each applied load the magnitude of half CMOD is noted from the $y$ displacement of the node at point A (Figure 3). A typical deformed specimen compared to un-deformed specimen is shown in Figure 3, which demonstrates the method of obtaining CMOD. The CMOD data obtained from FE results is then used to compute the magnitude of CTOD using a relation given in ASTM E1290[24] as:

\[
\delta = \frac{(CMOD) a}{r b} \]

(4)

Figure 4: Effect of $a/W$ on the variation of $J$ vs. $\sigma/\sigma_y$ for plane stress condition for A302 steel material.

![Image](https://example.com/image2.png)

**Figure 5:** Effect of $a/W$ on the variation of $J$ vs. $\sigma/\sigma_y$ for plane stress condition for HY80 steel material.

![Image](https://example.com/image3.png)

**Figure 6:** Variation of $J$ vs. $\sigma/\sigma_y$ for different state of stress condition for HY80 and A302 steel.

The figure clearly shows the magnitude of $J$ is higher for plane stress condition than plane strain condition. The nature of variation is found to be same in both the materials.

At each applied load the CMOD is noted from the $y$-displacement of the node at point A (Figure 1). A typical deformed specimen compared to un-deformed specimen is shown in Figure 3. The variation of CMOD against the normalized applied load $\sigma/\sigma_y$ for different state of stress for A302 and HY80 steel material are studied. A typical variation CMOD vs. $\sigma/\sigma_y$ for plane stress condition of A302 and HY80 steel material are shown in Figure 7 and 8. The results demonstrate that initially CMOD increases linearly and latter increases considerably and nonlinearly, which is attributed to the effect of extensive plastic deformation at crack tip.
The magnitude of linearity and nonlinearity is dependent on material, state of stress and a/W. In plane strain condition the linearity is observed more than the plane stress condition. The nature of variation is observed same in both the materials. As expected the magnitude of CMOD is more in case of plane stress condition than in plane strain condition. A typical variation of CMOD vs. $\sigma/\sigma_y$ for different state of stress condition for HY80 and A302 steel for a/W=0.50 is shown in Figure. 9. The CMOD data are used to compute the magnitude of CTOD for different state of stress and material using Eqn. 4, 5 and 6.

![Figure 7: Effect of a/W on the variation of CMOD vs. $\sigma/\sigma_y$ for plane stress condition for A302 steel material.](image)

![Figure 8: Effect of a/W on the variation of CMOD vs. $\sigma/\sigma_y$ for plane stress condition for HY80 steel material.](image)

![Figure 9: Variation of CMOD vs. $\sigma/\sigma_y$ for different state of stress condition for HY80 and A302 steel.](image)

A typical variation of CTOD vs. $J/\sigma_y$ for plane stress condition of A302 and HY80 steel material is shown in Figure. 10 and 11. From these figures it is clear that the variation of CTOD against $J/\sigma_y$ is non-linear; but if we neglect the elastic portion as, the variation is found to be linear. The variation of CTOD against $J/\sigma_y$ is dependent on a/W. The linearity portion of the plot is considered for the estimation of $d_n$. The magnitude of linearity and nonlinearity is dependent on material, state of stress and a/W.

A typical variation of CTOD vs. $J/\sigma_y$ for different state of stress condition for HY80 and A302 steel for a/W=0.50 is shown in Figure. 12. The nature of variation is observed to be same in both the materials. In the present investigation the constant $d_n$ in the relationship between $\delta$ and $J/\sigma_y$ (Eqn. 2) is obtained by the slopes of the linear portion of the CTOD and $J/\sigma_y$. The variation of a/W vs. $d_n$ for different state of stress condition for HY80 and A302 steel are studied and plotted in Figure. 13. The figure infers that the value of $d_n$ is less in plane strain than plane stress condition in both the materials. This figure clearly shows that the variation of $d_n$ obtained by both materials and state of stress are nonlinear with respect to variation in a/W ratio. It also indicates that there is considerable difference in magnitudes of $d_n$ obtained by both materials and state of stress in estimation of CTOD. The results in the Figure.13, indicate that for specimens with a/W < 0.50 the magnitudes of $d_n$ are higher, and is found to be >1 in both state of stress condition.

![Figure 10: Effect of a/W on the variation of CTOD vs. $J/\sigma_y$ for plane stress condition for A302 steel material.](image)

![Figure 11: Effect of a/W on the variation of CTOD vs. $J/\sigma_y$ for plane stress condition for HY80 steel material.](image)

The present results demonstrate that the relation between $J$ and CTOD strongly depends on the material, state of stress and a/W ratio of the specimens. The comparative study of variation of $d_n$ with a/W of HY80 and A302 steels with earlier results of IF steel [1] is also made and shown in Figure. 14. The figure indicates that the value of $d_n$ is much lower in case of IF steel than A302 and HY80 steels. It clearly infers the effect of yield stress on the variation of magnitude of $d_n$ with reference to a/W ratio as yield stress for IF, A302 and HY80 are 155MPa, 450MPa and 550MPA respectively. Kulkarni et al.[11] have shown that fracture analysis of thin sheets can be done using critical CTOD, $\delta_c$. In their
analysis, critical CTOD was computed by the relation between $J$ and $\delta$ suggested by Shih[6]. The thin sheet analysis is considered as fully plastic case and $d_p$ in Eqn. (1) is taken as unity. But, the present analysis infers that while converting magnitude of CTOD to $J$ one need to carefully evaluate the value of $d_p$ depending on the material, state of stress and a/W rather than considering it to be unity.

Figure 12: Variation of CTOD vs. $J/\sigma$ for different state of stress condition for HY80 and A302 steel.

Figure 13: Variation of a/W vs. $d_p$ for different state of stress condition for HY80 and A302 steel.

Figure 14: Variation of $d_p$ against a/W for plane stress condition for HY80 A302 and IF steel.

Conclusions

In this investigation the relationship between $J$ and CTOD are studied with respect to different materials, a/W and loading using 2D elastic-plastic finite element analysis. Following conclusions are drawn from the present investigation:

i. For the same applied load the intensity of $J$-integral depends on the a/W ratio of the specimen, material and state of stress.

ii. The variation of CTOD against $J/\sigma$ is highly dependent on a/W material and state of stress.

iii. The variation of $d_p$ obtained by different materials and state of stress are nonlinear with respect to variation in a/W ratio.

iv. While converting the magnitude of CTOD to $J$ one need to carefully evaluate the value of $d_p$ depending on the material property and a/W rather than considering it to be 1 in EPFM.

References