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# Fatigue Durability Assesment of Suspended Piping Systems Under Random Loads

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**Abstract**

Durability of deepwater marine risers is addressed with respect to potential fatigue failure due to random fluctuating loads produced due to random sea state. 3D Nonlinear dynamic analysis of riser is carried out in the time domain using finite element solver ABAQUS/Aqua. The analysis duly takes into account the non-linearities due to large deformation and time-wise variation of submergence, buoyancy, added mass and resultant hydrodynamic loading. In the present study application of structural concepts for the evaluation of the fatigue resistance of marine riser, including reliability technique has been presented. Alternative S-N and FM formulations of fatigue are investigated. Probabilistic fracture mechanics approach predicts the fatigue life of welded steel structure in the presence of cracks under random spectrum loading. It is based on a recently proposed bi-linear relationship to model fatigue crack growth and incorporates a failure criterion to describe the interaction between fracture and plastic collapse. Uncertainty modeling, especially on fatigue crack growth parameters, is undertaken with the aid of recently published data in support of the bi-linear crack growth relationship. Results pertaining to fatigue reliability and fatigue crack size evolution are presented using the Monte Carlo Simulation Technique, and emphasis is placed on a comparison between linear and bi-linear crack growth models. The bi-linear S-N curve and crack growth model are found to lead to higher fatigue life estimates and shows sensitivity to many other parameters in addition to the stress state of the component. These findings implicate inspection schemes for components of the marine structures to ensure minimization of the surprises due to wide scatter of the fatigue phenomenon in marine environment. Variations in system configuration, service life and coefficients of crack growth laws have been studied on the parametric basis. To study the influence of various random variables on riser safety, sensitivity analysis has been carried out.

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## Introduction

Fatigue damage becomes an important limit state for design of infrastructure because of ever increasing demand on performance of structures and introduction of lighter materials. Structures with significant dynamic loading are prone to develop fatigue damage during their service life, especially those structures whose utilization is to be extended beyond their initial defined design life. The aircraft industry was the first to incorporate fatigue as a design criterion, and later the other industries followed such as offshore, nuclear, steel bridge engineering and later in the shipping industry. In the later the use of high strength steel has led to the definition of an explicit fatigue limit state for design. Design criteria against failure are based on data obtained using S-N curve approach. The introduction and development of Fracture Mechanics (FM) and reliability based methods for crack growth assessment has significant substantial benefits and understanding of different parameters and corresponding uncertainties involved in the fatigue damage process.

S-N curves combined with the hypothesis of linear cumulative damage are applied to carryout the fatigue design check under variable amplitude loading, eg. NORSOK [12].beside adequate design against fatigue failure, inspection and repair can be used to increase the reliability in view of the crack growth. In order to guarantee the effect of inspection, more details about crack growth than provided by the S-N data are required. The prediction of the fatigue crack growth using the fracture mechanics describes crack growth physically and is able to calculate the fatigue life up to fracture accounting for possible inspection effects. This approach is thus more sophisticated and its application is wide spread. Across the globe with new challenges coming from new design for exploitation of resources from ocean environment, existing ageing infrastructure and development in computer simulation techniques, engineers, designers, asset owners and associated authorities are actively promoting the adoption of whole-life methodologies in order to undertake cost–effective structure management [16]. As marine structures are concerned, the work so far has addressed the fatigue life based on the traditional linear S–N curve approach and linear Fracture Mechanic approach [3, 4,20] . Since, an offshore structure is intended to remain on station for the life of oil field it will be exposed to a wide variety of hazards, with the potential for environmental damage, structural failure or damage to the material. As a part of design process, there are requirements of structural strength based on criteria referring to failure modes, such as rupture due to over loading, fatigue failures, buckling or even unstable fracture. Risk assessment is essential to regulate design so that the resulting structure is safe, serviceable and economical ensuring satisfactory performance during its design life period. As evidenced from similar applications in offshore structures [17,19,21] and other steel structures [5,8,9,16], one possible avenue for meeting these objectives is the implementation of

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fracture mechanics approach, in the light of load and response uncertainties, probabilistic modeling and reliability analysis. The work described in this paper is aimed at the development of a probabilistic methodology for the assessment of marine structures using bilinear S-N curve and Fracture Mechanics approaches. Compared to earlier studies mentioned above, limit state function based on bi-linear crack growth law is proposed incorporating the various random variables involved.

Statistical information on crack growth parameters, arising from research in support of the new code, is used herein for probabilistic modeling of random variables. Furthermore, the offshore structure live loading spectrum is obtained using structure geometry and statistical parameters of the sea. In this paper, particular emphasis is given to the comparative results obtained using linear and bi-linear models and their implications for fatigue. The sensitivity analysis is carried out to study the effect of various parameters on the reliability of marine risers. This comparison is particularly relevant in fatigue assessment of structures subjected to live load spectra containing a high number of low stress ranges.

## Failure criteria and probabilistic modelling for the fatigue and fracture limit state

Failure has been defined through the limit state function $g(\underline{z})$ which is negative or zero at failure, $\underline{Z}$ is the vector of basic variables describing loads, material properties, geometry variables, statistical estimate and model uncertainties. The probability of limit state violation is defined as :

$$P_f = P\left[g\left(\underline{z}\right) \leq 0\right] = \int_{g(\underline{z}) \leq 0} f_z\left(\underline{z}\right) d\underline{z} \qquad (1)$$

Where, $f\left(\underline{z}\right)$ is the joint probability density function of vector, $\underline{Z}$ which, is the product of the individual probability density functions of uncorrelated random variables.

### *S-N based failure function*

The S-N method is used routinely in the structural integrity assessment of offshore installation at the design stage and after the application of weld improvement technologies during in-service inspection. It predicts fatigue life using the endurance curve for the relevant weld category: it provide a relationship between hotspot stress range (S) and fatigue life (N). New design approaches based on S-N curves e.g BS 7608[2], NORSOK [12], include two slope curves to account for the effect of variable amplitude loading. Haibach [6] proposed an integrated approach also using fracture mechanics, and derived a two slope S-N curve. The first segment of the S-N curve extends until a limit of constant amplitude validity, whereas the lower segment was found to have slope 2m1-1, with m1 being the slope of the upper segment. However, the definition of slope and the position of knuckle of both slopes seem to be not clear in design codes. For instance, the BS 7608, DNV - RP-C203 [4] and NORSOK codes define the knuckles at 107 cycles in the S-N curves, whereas the IIW [7] sets this value at 5x 106 cycles. The BS 7608 assigns a slope of the upper segment plus 2; NORSOK recommends a fixed value of 5 and IIW recommends using 2m1-1. This may be an indication that the lower segment implies uncertainties difficult to be quantified.

The bilinear S-N model is given by the following equation:

$$N = \begin{cases} A_1 S^{-m_1} & S > S_x \\ A_2 S^{-m_2} & S \leq S_x \end{cases} \qquad (2)$$

Where, S is the stress range, m1,m2, A1,A2 are empirical constants; and N is the number of cycles to failure. At the intersection point or the knuckle point stress Sx and the corresponding number of cycles Nx are given by the expression,

$$S_x = 10^{\left\{\frac{\log A_1 - \log N_x}{m_1}\right\}} \qquad (3)$$

the component is subjected to cyclic loads, which are random in nature. Consequently the stress process is a stochastic process and each stress range is a random variable. The fatigue damage under staochastic loading has been calculated by the Miner-Palmgren model. In this model it is assumed that the damage on the structure per load cycle $D_j$ is constant at a given stress range $S_j$ and is given by

$$D_j = \frac{1}{N\left(S_j\right)} \qquad (4)$$

where $N\left(S_j\right)$ is the number of cycles to failure at a stress range $T_S$ The total damage accumulated in time $T_s$ is thus given by

$$D = \frac{1}{A} . \sum_{j=1}^{N(T_s)} \frac{1}{N\left(S_j\right)} p_j \qquad (5)$$

where $N(T_s)$ is the total number of stress cycles in time $T_s$. In this formulation it is assumed that the accumulated damage $D$ is independent of the sequence in which stress cycle occur.

Accumulated damage $D$ using the linear S-N curve is given as

$$D = \frac{N(T_s)}{A} \int_0^\infty S_j^m f_s\left(s\right) ds = \frac{N(T_s)}{A} E\left[S_j^m\right] \qquad (6)$$

Since each stress range is a random variable $\sum_{j=1}^{N(T_s)} S_j^m$ is also a random variable. If $N(T_s)$ is sufficiently large, the uncertainty in the sum is very small and the sum can be replaced by its expected value. Therefore

$$E\left|\sum_{j=1}^{N(T_s)} \frac{S_j^m}{A}\right| = E\left[N\left(T_s\right)\right] E\left[S_j^m\right] \qquad (7)$$

For a narrow band Gaussian process, stress ranges are Rayleigh distributed. The mean value of the stress range follows directly as.

$$E\left[S_j^m\right] = \int_0^\infty \left(2x\right)^m \frac{x}{\sigma_x} \exp\left(-\frac{1}{2}\left(\frac{x}{\sigma_x}\right)^2\right) dx = \left(2\sqrt{2}\right)^m \sigma_x^m \Gamma\left(1 + \frac{m}{2}\right) \qquad (8)$$

 

The accumulated damage using bilinear S-N curve approach is given as

$$D = \frac{N(T_s)}{A_1}\int_0^{S_x} S_J^{m_2} f_s(s)\,ds + \frac{N(T_s)}{A_2}\int_{S_x}^{\infty} S_J^{m_1} f_s(s)\,ds \tag{9}$$

Considering that the long-term stress acting on the riser is induced by environmental loadings that can be described as a set of stationary short term sea states, the total fatigue damage can be obtained by summing the accumulated damage over all sea states. The total damage D in general may be written as:

$$D = \frac{T_s}{A}\Omega, \tag{10}$$

Where $\Omega$ is a stress parameter and for bilinear model it is given by equation (11).

$$\Omega = (2\sqrt{2})^{m_1}\Gamma\left(1+\frac{m_1}{2};\left(\frac{S_x}{2\sqrt{2}\sigma}\right)^2\right)\sum_{q=1}^{n} f_q v_{o_q}\sigma_q^{m_1} + (2\sqrt{2})^{m_2}\Upsilon\left(1+\frac{m_2}{2};\left(\frac{S_x}{2\sqrt{2}\sigma}\right)^2\right)\sum_{q=1}^{n} f_q v_{o_q}\sigma_q^{m_2} \tag{11}$$

where, m1,m2 are the material constants based on the experimental data. where $f_q$ fraction of time in qth sea state, $v_{0_q}$ mean zero crossing frequency of random stress process in qth sea state, $\sigma_q$ is R.M.S of stress process in qth sea state. $\Gamma$ is the complimentary incomplete gamma function and $\Upsilon$ is the incomplete gamma function.

In the above expression $T_s$ is years in service, $v_{0_q} = \frac{1}{2\pi}\sqrt{m_2/m_o}$, is zero crossing frequency of the stress process in qth sea state, $\sigma_q = \sqrt{m_o}$, is the RMS value of the stress process in qth sea state. $m_q = \int_0^{\infty}\omega^n S(\omega)\,d\omega$ is the nth moment of the stress spectrum and $f_q$ is fraction of the time spent in the qth sea state( to account for long term sea effect).

According to Miner's rule, fatigue failure occurs if $D > \Delta_F$, where $\Delta_F$ is the critical cumulative damage, which is often taken as 1. Letting $D = \Delta_F$, the basic damage expression of equation (10) can be expresses in terms of time to failure,

$$T = \frac{\Delta_F . A}{\Omega} \tag{12}$$

In order to take into account the uncertainties associated with the above expression, the factors involved in the expression shall be modeled as random variables. The time to failure $T_i$ in general may be given as

$$T_i = \frac{\Delta_F . A_i}{B_i^m \Omega_i} \tag{13}$$

where, $\Delta_F$, $A_i$, $B_i$ are random variables.

In the above eq , $B_i$ describes the inaccuracies in estimating the fatigue stresses. The actual stress range is assumed to be equal to the product of $B_i$ and the estimated stress range S. the uncertainties in fatigue strength, as evidenced by the scatter in S-N data, are accounted by considering $A_i$ to be random variable. The random variable $\Delta_F$ quantifies modeling error associated with the Miner-Palmgren rule.

The fatigue failure occurs when the random variable $T_i$ is smaller than $T_s$ where $T_s$ is the lifetime of the structure. Thus, the limit state function for bilinear fatigue model is

$$g(\underline{z}) = \frac{\Delta_F A_i}{B_i^m \Omega_i} - T_s \tag{14}$$

*Fracture Mechanics based failure function*

The fatigue reliability and updating formulation used is based on a fracture mechanics approach given by the Paris crack propagation law (e.g G Madsen et al.[10]; Lotsberg and Sigurdsson[15]; Moan et al.[11]). In general the Paris law may be used in a multi segmented crack growth as follows:

$$\frac{da}{dN} = \begin{cases} 0 \; for \Delta K \le \Delta K_o \\ A_1(\Delta K)^{m_1} \; for \Delta K_o < \Delta K < \Delta K_1 \\ \vdots \\ A_{n+1}(\Delta K)^{m_{n+1}} \; for \Delta K > \Delta K_n \end{cases} \tag{15}$$

where, a is the crack depth, N is the number of cycles, Ai is the crack growth rate parameter for segment i and m is its corresponding slope, $\Delta K_{th}$ is the threshold of the stress intensity factor range and $\Delta K_i$ is the point of intersection of two consecutive segments. In the present study bilinear elastic fracture mechanics has been adopted for limit state function as recommended by BS7910 [1] for the fatigue assessment of welded structures, it is based on the study carried out by King [5]. King performed a comprehensive collection of data from different sources and recommends a two segment crack growth law for steels. The uncertainties reported for both the segments are different, with the largest variability in the near threshold segment due to inherent uncertainty in $\Delta K$, threshold below which no growth is experienced. On the other hand lower uncertainty of the upper segment corresponds to the crack growth rates behavior well inside the stable region with higher values of $\Delta K$. The two segments of the crack growth law are assumed as uncorrelated.

In this approach, relationships between average increment in crack growth $\left(\frac{da}{dN}\right)$ during a load cycle and a global parameter are developed. The most popular global parameter used is the stress intensity factor, K, which gives the magnitude of stresses in the crack tip region as a function of type and magnitude of loading and geometry of the cracked body. $\Delta K$ is usually expressed as:

$$\Delta K = \Delta S Y(a)\sqrt{\pi a} \tag{16}$$

 

where, a is the crack size; S is the far-field stress due to applied load; Y(a) is the geometry function which takes into account crack geometry and specimen shape.

In the present study an empirical expression for Y (a) is given as under.

$$Y(a) = C_r a^k \tag{17}$$

Where Cr = 1(constt.), a=-1.025 [6]

Under variable amplitude loading and for a bilinear crack growth model operating on a single degree of freedom crack with a single crack dimension 'a', a first order approximation to the average number of cycles to failure through equation (2) leads to:

$$\begin{aligned} &E\left[N\left|\left(a_{in} \le E[a_{th}]\right)\right.\right] = \infty \\ &E\left[N\left|\left(E[a_{th}] < a_{in} < E[a_{tr}]\right)\right.\right] \\ &= \sum_{i=1,2} \frac{1}{A_i} \int_{a_{in,i}}^{a_{f,i}} \frac{da}{E[\Delta S]^{m_i} \left(Y(a)\sqrt{\pi a}\right)^{m_i}} \end{aligned} \tag{18}$$

where, $a_{tr}$ crack depth at the transition or intersection, $a_f$ crack depth at the failure.

The first part of the equation (18) states that if the initial crack depth $a_{in}$, that is the crack depth at the start of the fatigue process, is less than the stress spectrum, threshold crack depth, the fatigue life is infinite. However, since fatigue crack growth can be triggered by stress crossing the threshold stress range, $E[a_{th}]$ is determined from the maximum stress value in the stress range. The second part of the equation (18) states that if $a_{in}$ gives rise to near threshold crack growth, the crack will grow in the threshold region (crack growth parameters $A_1$ and $m_1$) from $a_{in,1} = a_{in}$ to $a_{f,1} = E[a_{tr}]$. From this crack depth the crack will grow in the Paris region (crack growth parameters $A_2$ and $m_2$) from $a_{in,2} = a_{f,1} = E[a_{tr}]$ to $a_{f,2} = a_{fail}$, where $a_{fail}$ is the maximum crack depth that may be sustained by the section. In the present study it is taken as the thickness of the pipe.

The second part of equation (18) can also be represented in more general form in terms of crack size and number of cycles in time Ts as

$$\begin{aligned} &\frac{1}{A_1} \int_{a_{in}}^{a_{f,1}} \frac{dz}{[Y(z)]^{m_1} \left(\sqrt{\pi z}\right)^{m_1}} + \frac{1}{A_2} \int_{a_{f,1}}^{a_{fail}} \frac{dz}{[Y(z)]^{m_2} \left(\sqrt{\pi z}\right)^{m_2}} \\ &= N(T_s)E(\Delta S^{m_1}) + N(T_s)E(\Delta S^{m_2}) \end{aligned} \tag{19}$$

If $N(T_s)$ is sufficiently large, the uncertainty in the sum is very small and it can be replaced by its expected value. This approximation neglects the effects of load cycle sequence. Assuming the environmental loading condition being described as a long term sea states and the stress range following a Rayleigh distribution in each sea state, we obtain

$$\begin{aligned} &\frac{1}{A_1} \int_{a_{in}}^{a_{f,1}} \frac{dz}{[Y(z)]^{m_1} \left(\sqrt{\pi z}\right)^{m_1}} + \frac{1}{A_2} \int_{a_{f,1}}^{a_{fail}} \frac{dz}{[Y(z)]^{m_2} \left(\sqrt{\pi z}\right)^{m_2}} \\ &= T_s \Omega \end{aligned} \tag{20}$$

Where $\Omega$ is a stress parameter and it is given by equation (21).

$$\begin{aligned} \Omega = &(2\sqrt{2})^{m_1} \Gamma\left(1 + \frac{m_1}{2}; \left(\frac{\Delta K_{th}}{2\sqrt{2}}\right)^2\right) \sum_{q=1}^{n} f_q v_{o_q} \sigma_q^{m_1} + \\ &(2\sqrt{2})^{m_2} \Upsilon\left(1 + \frac{m_2}{2}; \left(\frac{\Delta K_{tr}}{2\sqrt{2}}\right)^2\right) \sum_{q=1}^{n} f_q v_{o_q} \sigma_q^{m_2} \end{aligned} \tag{21}$$

where, A1, A2, m1,m2 are the material constants based on the experimental data. $\Delta K_{th}$ is the threshold stress intensity range, $\Delta K_{tr}$ is the stress intensity factor at the intersection, $v_{o_q} = \frac{1}{2\pi}\sqrt{\frac{m_2}{m_0}}$ is the zero crossing frequency of the stress process in the qth sea state, $\sigma_q = \sqrt{m_0}$ is the RMS value of the stress process in the qth sea state. $m_q = \int_0^\infty \omega^n S(\omega) d\omega$ $n$th moment of stress spectrum and $f_q$ is a fraction of time spent in qth sea state( to account for long term sea effect). $\Gamma$ is the complimentary incomplete gamma function and $\Upsilon$ is the incomplete gamma function.

The failure criteria can then be formulated as a function of crack size. Failure occurs when the crack size exceeds a critical value "$a_{fail}$" which is based on serviceability conditions. The probabilistic model in general for the time to failure $T_i$ of joint 'i' is defined as follows, taking into account the uncertainties involved in the fracture mechanics model:

$$T_i = \frac{1}{A_i B_i^m \Omega_i} \int_{a_{in}}^{a_{fail}} \frac{dz}{\left[\gamma_{geom} Y(z)\right]^m \left(\sqrt{\pi z}\right)^m} \tag{22}$$

Where A, B, ain and $\gamma_{geom}$ are the random variables. Here B and $\gamma_{geom}$ were introduced to model errors in the estimation of the stress range ΔS and in the geometry function Y (a), respectively. The fatigue failure occurs when the random variable Ti is smaller than Ts, where Ts is the lifetime of the structure. Thus the limit state function is

$$g(z) = \frac{1}{A_i B_i^m \Omega_i} \int_{a_{in}}^{a_{fail}} \frac{dz}{\left[\gamma_{geom} Y(z)\right]^{m_i} \left(\sqrt{\pi z}\right)^{m_i}} - T_s \tag{23}$$

The surface g (z) = 0 is the limit state surface, and z is the vector of basic random variables in the problem constituting the limit state function g (z).

The probability of failure Pf ,

$$P_f = P(T \le T_s) = P[g(z) \le 0] \quad (24)$$

The failure probability can be computed using FORM and Monte Carlo Simulation technique. The reliability or safety index( $\beta$ ) is thus obtained by

$$\beta = \Phi^{-1}(P_f) \quad (25)$$

Where $\Phi^{-1}$ is the inverse of the standardized normal distribution function.

## Numerical study

For the numerical study, a marine riser shown in Fig.1 is chosen for the reliability study. A non-linear 3D dynamic analysis of marine riser has been carried out in time domain using finite element package ABAQUS/Aqua. The riser is modeled as tensioned beam with six degrees of freedom at each node (three translation and three rotations). The bottom end of the riser is hinged and assumed to be restrained in horizontal and vertical directions; the top end of the riser is restrained in the horizontal direction. The analysis includes nonlinearities due to large deformation, time-wise variation of submergence, buoyancy, added mass and drag force. More detailed analysis is given in [14]. Its specifications are given in Table 1. For the reliability analysis we need the response statistic of riser stresses arising due to the action of environmental forces. For the present study required the riser statistics are shown in Table 2 and 3 for random sea states. The random variables considered in the reliability study for the S-N model and Fracture Mechanics model are summarized in Table 4, 5 and 6. Monte Carlo Simulation technique has been used to carry out the reliability analysis.

**Table 1:** Marine Riser Specifications

| | |
|---|---|
| Riser Tension | $=3 \times 10^6$ N |
| Water depth | =500 m |
| Riser length | =500 m |
| Outer diameter of riser | =0.406 m |
| Inner diameter of riser | =0.374 m |
| Mass density of steel | $=7840$ kg/m$^3$ |
| Mass density of water | $=1025$ kg/m$^3$ |
| Coefficient of drag, Cd | =1.1 |
| Coefficient of Inertia, Cm | =2.5 |
| Modulus of elasticity | $=2.1 \times 10^{11}$ N/m$^2$ |
| Service life | =20 years |

**Table 2** :Simulated Sea States

| Sea States | Significant wave height $H_s$(m) | Zero crossing period $T_z$ (s) | Wind velocity U (m/s) | Fraction of time in each sea state |
|---|---|---|---|---|
| S1 | 17.15 | 13.26 | 24.38 | 0.00000036 |
| S2 | 15.65 | 12.66 | 23.29 | 0.00000237 |
| S3 | 14.15 | 12.04 | 22.15 | 0.00001426 |
| S4 | 12.65 | 11.39 | 20.94 | 0.00007921 |
| S5 | 11.15 | 10.69 | 19.66 | 0.00040274 |
| S6 | 9.65 | 9.94 | 18.29 | 0.00185765 |
| S7 | 8.15 | 9.14 | 16.81 | 0.00768243 |
| S8 | 6.65 | 8.26 | 15.18 | 0.02802024 |
| S9 | 5.15 | 7.26 | 13.36 | 0.08789058 |
| S10 | 3.65 | 6.12 | 11.25 | 0.22674599 |
| S11 | 2.15 | 4.69 | 8.63 | 0.43255715 |
| S12 | 0.65 | 2.58 | 4.75 | 0.21474688 |

**Table 3:** Statistics of random stress response (only under waves)

| Sea State | Fraction of time in each sea state | RMS Stress (MPa) | Zero crossing rate (Hz) |
|---|---|---|---|
| S1 | 0.00000036 | 11.83 | 0.089 |
| S2 | 0.00000237 | 10.84 | 0.091 |
| S3 | 0.00001426 | 9.84 | 0.094 |
| S4 | 0.00007921 | 8.21 | 0.100 |
| S5 | 0.00040274 | 7.34 | 0.104 |
| S6 | 0.00185765 | 6.66 | 0.108 |
| S7 | 0.00768243 | 6.56 | 0.114 |
| S8 | 0.02802024 | 5.57 | 0.121 |
| S9 | 0.08789058 | 4.88 | 0.131 |
| S10 | 0.22674599 | 4.53 | 0.142 |
| S11 | 0.43255715 | 2.35 | 0.162 |
| S12 | 0.21474688 | 1.95 | 0.173 |

**Table 4** :Data for reliability study Fracture

| Variable | Distribution | Mean/Median | COV |
|---|---|---|---|
| Paris coefficient, *A* | Lognormal | $\tilde{A}=1.8\times10^{-12}$ | 0.55 |
| Stress modeling error, *B* | Lognormal | $\tilde{B}=1.00$ | 0.20 |
| Initial crack length $a_o$ (mm) | Exponential | $\mu_{a_0}=0.18$ | |
| Paris exponent, *m* | Constant | 3.0 | |
| Modeling error in *Y(a)* $\gamma_i$ | Lognormal | $\mu_\gamma=1.00$ | 0.10 |
| Critical crack length $a_c$ (mm) | Constant | 16.0 | |

~ = Median value; $\mu$ = Mean value; COV= Coefficient of variation

**Table 5** Data for reliability study Fracture Mechanics model (Bi-linear)

| Variable | Distribution | Mean/Median | COV |
|---|---|---|---|
| Paris coefficient, $A_1$ | Lognormal | $\tilde{A}_1=4.8\times10^{-18}$ | 1.69 |
| Stress modeling error, *B* | Lognormal | $\tilde{B}=1.00$ | 0.20 |
| Initial crack length $a_o$ (mm) | Exponential | $\mu_{a_0}=0.18$ | |
| Paris exponent, $m_1$ | Constant | 5.1 | |
| Modeling error in *Y(a)* $\gamma_i$ | Lognormal | $\mu_\gamma=1.00$ | 0.10 |
| Critical crack length $a_c$ (mm) | Constant | 16.0 | |
| Paris coefficient, $A_2$ | Lognormal | $\tilde{A}_2=6.0\times10^{-12}$ | 1.18 |
| Paris exponent, $m_2$ | Constant | 2.67 | |

~ = Median value; $\mu$ = Mean value; COV= Coefficient of variation

**Table 6:** Data for reliability study S-N model (Linear &Bi-linear)[4]

| Variable | Distribution | Mean/Median | COV |
|---|---|---|---|
| Fatigue strength coefficient, $A_1$ | Lognormal | $\tilde{A}_1=1.56\times10^{12}$ | 1.69 |
| Stress modeling error, *B* | Lognormal | $\tilde{B}=1.00$ | 0.20 |
| Miner Palgren damage index $\Delta_F$ | Lognormal | $\tilde{\Delta}_F=1.00$ | 0.30 |
| Fatigue exponent, $m_1$ | Constant | 3.0 | |
| Fatigue Strength coefficient, $A_2$ | Lognormal | $\tilde{A}_2=2.08\times10^{16}$ | 1.18 |
| Fatigue exponent, $m_2$ | Constant | 5.0 | |

~ = Median value; $\mu$ = Mean value; COV= Coefficient of variation

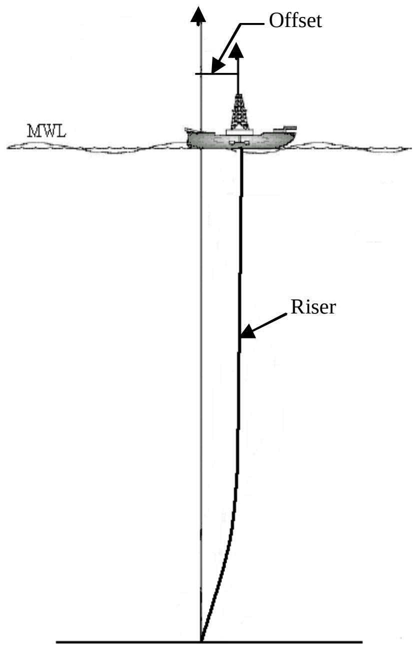


**Figure 1:** Typical Marine Riser

## Results and Discussion

To study the effect of linear and bilinear S-N curve, Paris crack growth law and bilinear crack growth law the following studies has been carried out.

*(a) Comparison of reliability indices for variation in system configuration.*

The number of joints and initial crack length has been varied and comparison of Reliability levels for linear and bilinear S-N curve and FM linear and bi-linear models for variations in number of joints is shown in Figure 2, 3 and 4. As evident from the graph that higher reliability index are exhibited for the bi-linear law and the linear results are on the conservative side.

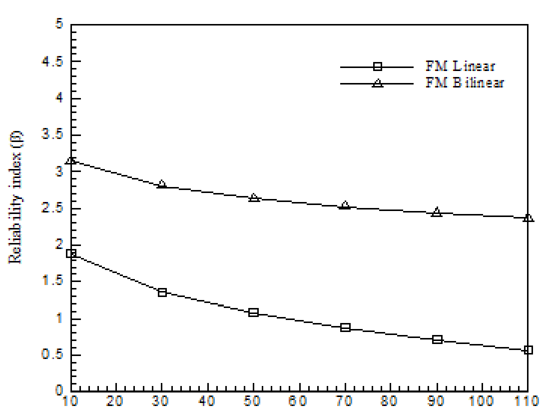


**Figure 2**: Comparison of reliability levels against variations in number of joints

The initial crack size of a surface crack in welded joints induced by the welding process itself is of interest herein. The initial crack size may be defined as the depth from which a flaw which has nucleated from surface defects and will grow under the stable crack propagation until a final crack size or fracture. Reliability based crack growth analysis is very sensitive to the mean value of initial crack size. Figure 5 shows the comparison of FM linear and Bilinear Curves for variation in Initial crack length (mm). It shows that bilinear results exhibit higher reliability indices and the linear results are on the conservative side.

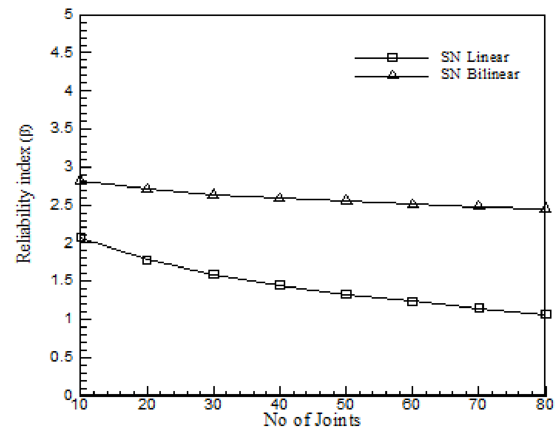


**Figure 3**: Comparison of reliability levels against variations in number of joints

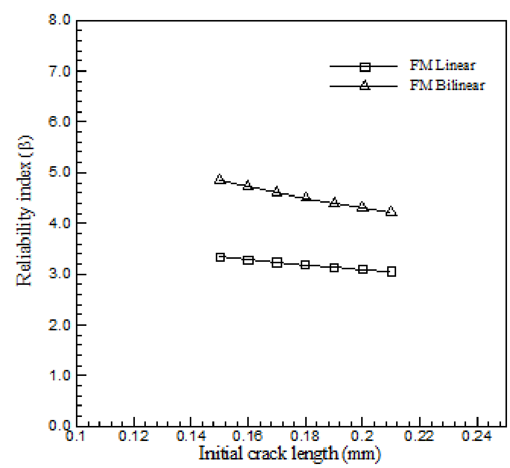


**Figure 4**: Comparison of reliability levels against variations in initial crack length (mm)

*(b) Comparison of reliability indices for change in material parameters*

Comparison of reliability indices for change in empirical constants *m & A* of fracture mechanics model are shown in figures 5. Variation in Paris coefficient (figure 5) shows clearly better results for the bi-linear model; however the variation in Paris exponent and equivalent in bilinear law does not yield any significant difference. Figure 6 shows the effect of variation in Miner Palmgren damage index. It shows that the reliability is increases with the increase in mean value of the Damage index. Figure 7 shows that there is increase in value of reliability index as the value of Fatigue strength coefficient increases. It's clearly evident that the bilinear results exhibits higher reliability index and the linear results are on the conservative side.

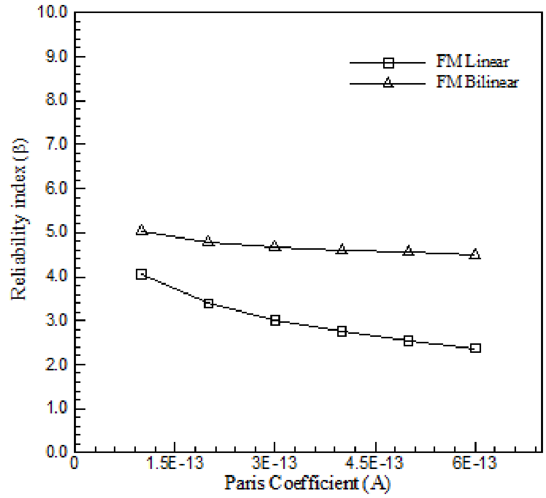


**Figure 5**: Comparison of reliability levels against variations in Paris coefficient (A)

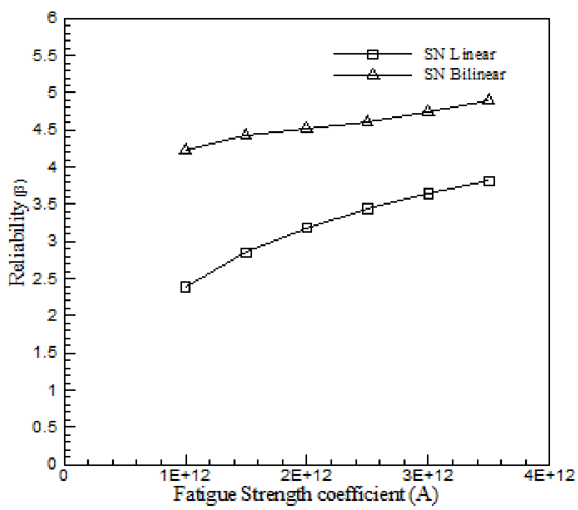


**Figure 6**: Comparison of reliability levels against fatigue strength coefficient (A)

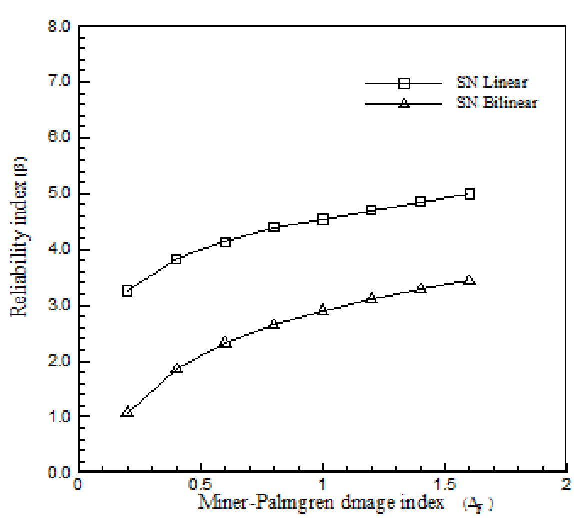


**Figure 7**: Comparison of reliability levels against variations in Miner Palmgren index

*Sensitivity analysis*

This analysis has been carried out to study the influence of various random variables on riser reliability. This influence is measured in terms of sensitivity factor ($\alpha_j$), which for the *j*th random variable is defined as Madsen et al.[10]:

$$\alpha_j = \frac{\partial \beta}{\partial y_j} \quad \Bigg| \quad \underline{y}^* = \frac{y_j^*}{\beta} \qquad (26)$$

where $\underline{y}^*$ = point minimizing Eq. (26), usually referred to as design point, and $y_j^*$ = value of the *j*th random variable at this point.

The above defined sensitivity factors have the following characteristics:

1. The lower the magnitude of $\alpha_j$, less is the influence of the *jth* random variable on the reliability.
2. $\alpha_j$ is positive for the load variables and negative for resistance variables.
3. If $\alpha_1, \alpha_2, \alpha_3 \ldots\ldots\ldots\ldots \alpha_n$ are the sensitivity factors for *n* random variables appearing in the limit state function then $\sum_{j=1}^{n} \alpha_j^2 = 1$

In the present study, using above expression, sensitivity factors for each random variables have been determined and shown graphically in Fig.8. As mentioned above, the magnitude of this factor for a random variable is directly measure of its influence on riser reliability. However, its sign determine whether the random variable is a load variable or resistance variable. The negative value of the sensitivity factor indicates that the random variable is a resistance variable i.e., its increase will improve the riser reliability and decrease will reduce the reliability. Similarly positive value of sensitivity factor indicates that it is a load variable and its influence would be opposite to that of a resistance variable. The major advantage of this study is that without carrying out any separate parametric study for each variable one can directly know how a particular random variable affects the riser reliability. Fig 3 shows the results of sensitivity analysis for S-N curve based model. The bar chart indicates that the sensitivity factors for Miner-Palmgren damage index ($\Delta$), and fatigue strength coefficient (*A*) are negative, hence, they are resistance variables and contribute to resistance part of limit state function. Sensitivity for stress modeling error or response uncertainty factor (*B*), however, is positive then it will contribute to load part of the limit state function. Therefore for the given uncertainty an increase in the magnitude of Miner-Palmgren damage index ($\Delta$) and fatigue strength coefficient (*A*) will improve the reliability of risers. Moreover, the chart shows that out of the two resistance variables, reliability is more sensitive to fatigue strength coefficient (*A*) than Miner-Palmgren damage index ($\Delta$).

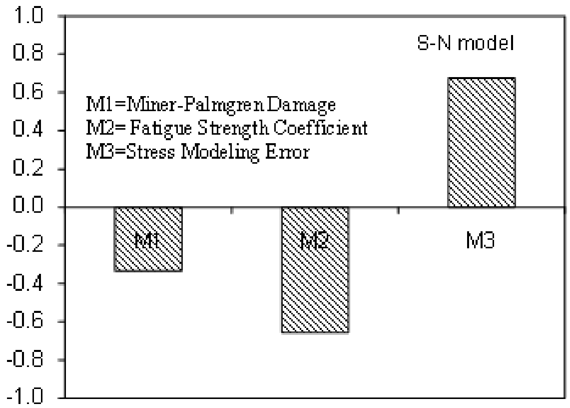


**Figure 8**: Sensitivity diagram

## Conclusions

This paper has outlined a probabilistic Fatigue and fracture mechanics approach to predict the fatigue life of suspended piping systems in the presence of cracks under random environmental loading on an offshore structure (marine riser).Limit state function for both S-N Curve and fracture mechanics based approaches incorporating all random variables have been derived. Results pertaining to fatigue reliability and crack size evolution and other parameters affecting the fatigue reliability are presented and emphasis is placed on a comparison between linear and bi-linear S-N curve & crack growth models. The latter is found to lead to higher fatigue life estimates, which have implications on inspection schemes for offshore structures. These differences are worth noting in the case of offshore structures, especially for cases where stress range spectra contains high number of low amplitude cycles. However, the complex and highly random nature of the fatigue process implies that any procedure should be used cautiously and, wherever possible, should be complimented with the inspection data if available. The sensitivity analysis shows the importance of various parameters and its effect on the reliability of marine risers.

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