



## Study of the Influence of Module on Fatigue Life of Spur Gear

Amitesh. B and Ajit Prasad. S.L

Department of Mechanical Engineering, P.E.S College of Engineering, Mandya-571401, India.

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### Abstract

Spur gears are most commonly used transmission elements in engineering applications. Two types of fatigue failure occur in spur gears i.e., bending fatigue and contact fatigue. Bending fatigue leads to breakage of gear teeth, contact fatigue is a surface fatigue failure like pitting, scoring etc.,. In the present work, bending fatigue failure is analyzed and fatigue life is determined based on both stress life & strain life approach. The stresses developed at the root of gear teeth due to the moving load along the tooth flank is determined theoretically, considering the effect of contact ratio. Fatigue life of the gear is estimated theoretically, for particular operating conditions of power, centre distance, speed ratio and face width, considering different values of module. The calculated values of root stresses and fatigue life are compared with the results obtained by numerical analysis, using standard FEA tool and are found to be matching within reasonable limits.

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### Introduction

Spur gears are most commonly used transmission elements in engineering applications. Gear teeth are subjected to fluctuating load and maximum stress occurs at the root of the gear tooth because of maximum bending moment and stress concentration effect. Load acting on a gear tooth doesn't remain constant during the period of contact. Gear tooth load changes with the moving point of contact. Tooth load pattern during contact period is also influenced by the contact ratio between the meshing gears.

Fluctuating load results in tooth failure due to fatigue and fatigue life is crucial in design of gears. Influence of module on fatigue life of gear teeth has been studied in the present work keeping all other parameters constant.

### Objective

The main objectives of the present work are (1) Determination of stress at gear tooth root considering the moving load along the tooth flank and contact ratio. (2) Study of influence of module on root stress. (3) Estimation of fatigue life of gear teeth by stress-life and strain-life approach.

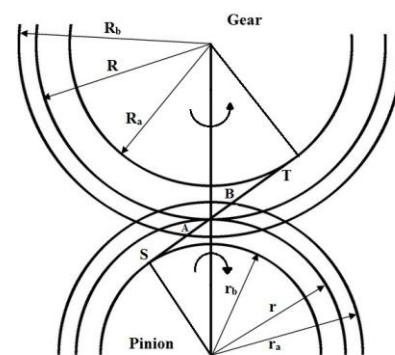
### Gear tooth load

The path of contact for a pair of spur gear teeth is shown in Fig. 1(a), with contact beginning at 'A' and ending at 'B'. During this contact period the point of application of load moves along the flank surfaces of the mating teeth as indicated in Fig. 1(b). For constant torque transmission, load acting at the point of contact changes continuously during the contact period, resulting in variation of bending stress induced at the root of gear tooth<sup>1</sup>.

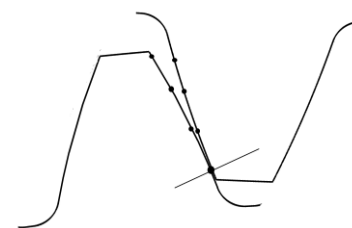
### Effect of contact ratio

For smooth transmission of power, the contact ratio between mating gears will be normally greater than '1'. i.e., For certain period of the contact, two pairs of teeth will be sharing the load,

while for the remaining period, only one pair of teeth will be taking the entire load. Fig. 2 illustrates typical load distribution<sup>2</sup> during contact period, for contact ratio between 1 & 2. Thus, by considering the effects of moving tooth load and contact ratio, the stresses at the tooth root will be fluctuating continuously from '0' to maximum and to '0', as indicated in Fig. 3.



(a)



(b)

Figure 1: (a) Gear Terminology (b) Moving Load

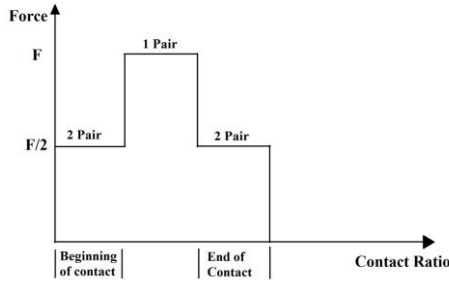


Figure 2: Load distribution factor

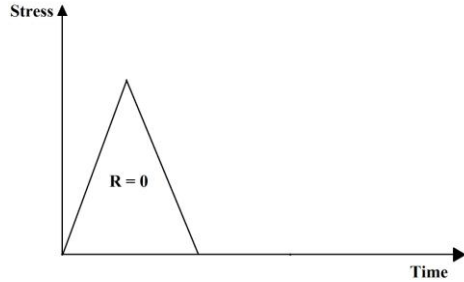


Figure 3: stress cycle for fatigue analysis

The stress at the root of the gear tooth is also affected by stress concentration factor at the root fillet<sup>3</sup>. Stress concentration factor at the root is determined by equation (1), where t = thickness of tooth at the weakest section, r = minimum fillet radius, h = height of applied load above the weakest section.

$$Kt = 0.18 + \left[ \left( \frac{t}{r} \right)^{0.15} * \left( \frac{t}{h} \right)^{0.45} \right] \dots\dots(1)$$

**Fatigue Analysis**

Fatigue occurs when a material is subjected to repeated loading. When the loads are above a certain threshold, microscopic cracks will begin to form at the stress concentrators such as fillets, bolt holes etc., once a micro crack initiates, it starts to propagate and reaches to the critical size and fracture takes place. The fatigue life can be estimated by two approaches:

- (1) Stress-life approach
- (2) Strain-life approach

**Stress-life approach**

It is a stress based model, which is used often for high cycle fatigue. Goodman's equation (2) and Basquin equation (3) are used for fatigue life estimation by stress-life approach.

$$\left( \frac{\sigma_a}{\sigma_{Nf}} \right) + \left( \frac{\sigma_m}{\sigma_{ut}} \right) = 1 \dots\dots(2)$$

$$\sigma_{Nf} = aN^b \dots\dots(3)$$

Where  $\sigma_a$  is the alternating stress,  $\sigma_{Nf}$  is the fatigue strength,  $\sigma_m$  is the mean stress,  $\sigma_{ut}$  is the ultimate tensile stress, N is the life in number of cycles, a is the co-efficient representing the value of  $\sigma_a$ , B is the slope.

**Strain-life approach**

It is a strain based model, which is used often for both low cycle and high cycle fatigue. The cyclic stress-strain curve is obtained by

using the equation (4). The expected fatigue life is based on the nucleation of small macro cracks. The fatigue crack initiation<sup>4</sup> is estimated by using the Coffin-Manson relation, where the total strain is the sum of elastic and plastic strain components shown in equation (5).

$$\epsilon_a = \left[ \frac{\sigma_a}{E} \right] + \left( \frac{\sigma_a}{K'} \right)^{1/n'} \dots\dots(4)$$

$$\epsilon_a = \left[ \frac{\sigma_f'}{E} (2N_f)^b \right] + \epsilon_f' (2N_f)^c \dots\dots(5)$$

Where  $\epsilon_a$  is the strain amplitude,  $\sigma_a$  is the stress amplitude,  $K'$  is the cyclic strength co-efficient,  $n'$  is the cyclic hardening co-efficient,  $\sigma_f'$  is the fatigue strength co-efficient, b is the fatigue strength exponent,  $\epsilon_f'$  is the fatigue ductility co-efficient, c is the fatigue ductility exponent, E is the Young's modulus.

**Selection of gear**

Gears are designed for power transmission applications based on operational parameters like power transmitted, speed ratios and shaft centre distance. Gears of different modules can be selected for identical operating parameters. In the present work, influence of module on root stress and fatigue life has been studied, considering identical operating parameters, face width and gear material.

**Experimental**

In order to fulfil the selected objectives, typical operating parameters are selected as follows:

- Power transmitted = P = 120 KW
- Speed/Gear ratio = G = 2
- Speed of the pinion = 650 Rpm
- Pressure angle =  $\phi = 20^\circ$
- Centre distance = C = 300 mm
- Face width = 'B' = 60 m
- Gear material = Steel 1010<sup>5</sup>
- $\sigma_{ut} = 331 \text{ MPa}$ ,  $\sigma_y = 200 \text{ MPa}$ ,  $E = 203 \text{ GPa}$
- $\sigma_f = 499 \text{ MPa}$ ,  $\epsilon_f' = 0.104$ ,  $b = -0.1$ ,  $c = -0.408$ ,  $K' = 867$ ,  $n' = 0.244$

For the above mentioned operating parameters, gears of different modules are selected and standard gear specifications are shown in Table 1.

Table 1: Gear specifications

Sl. No	Module (mm)	No. of teeth on Pinion $Z_1$	No. of teeth on gear $Z_2$	Contact ratio	Root tooth thickness (mm)	Mass of gear (Kg)	Stress concentration factor
1	3	67	134	1.84	7.10	55.40	1.52
2	4	50	100	1.80	9.30	55.34	1.50
3	5	40	80	1.77	11.42	55.29	1.48
4	6	34	67	1.74	13.56	55.27	1.47
5	7	29	57	1.71	15.41	55.23	1.45
6	8	25	50	1.68	17.19	55.22	1.43
7	9	23	45	1.66	18.92	55.21	1.42
8	10	20	40	1.64	20.36	55.19	1.40

The maximum value of bending stress at the root at the gear teeth is calculated using bending equation, for different modules of gears. For the selected gears and operating parameters, the stress cycles are calculated and fatigue life estimation based on stress-life and strain-life approach has been carried out based on Goodman and Coffin-Manson relation.

**Numerical Analysis**

The stress analysis and fatigue life estimation of the selected gears has also been carried out using FEA tool ANSYS.

The 3D model of spur gear is created in CATIA as shown in Fig. 4(a) and it is imported to ANSYS for the analysis. The hex dominant mesh is applied to the model as shown in Fig. 4(b). The boundary conditions are applied to the model i.e., the shaft hole is fixed and the nodal force is applied on the gear tooth as shown in Fig. 4(c). The model is solved for results and the equivalent stress obtained at the root of the gear tooth is shown in Fig. 4(d).

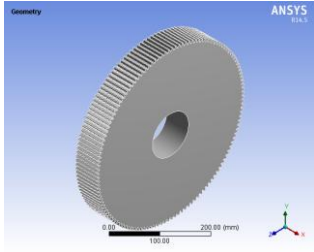


Figure 4(a): Spur Gear

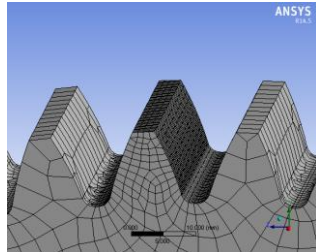


Figure 4(b): Hex dominant mesh

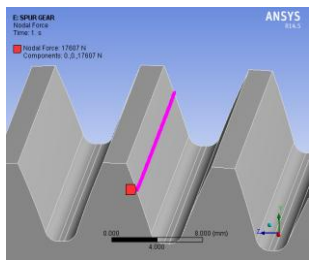


Figure 4(c): Nodal force

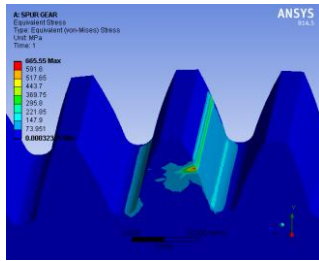


Figure 4(d): Equivalent stress

The fatigue analysis is carried out by both stress-life and strain-life approach. The stress-life approach is performed by using a fatigue tool. The type of loading – zero based stress, Mean stress theory – Goodman’s approach is selected, and the equivalent stress is considered for the stress component, the alternating stress is applied for the corresponding number of cycles. The model is solved for results and the life of the spur gear determined is shown in Fig. 5.

The strain-life approach<sup>6-7</sup> is performed by using a fatigue tool. The type of loading – zero based stress, Mean stress theory – Morrow’s approach is selected, and the equivalent stress is considered for the stress component. The strain-life parameters like fatigue strength co-efficient, fatigue strength exponent, fatigue ductility co-efficient, fatigue ductility exponent is applied. The model is solved for results and the life of the spur gear is determined as shown in Fig. 6.

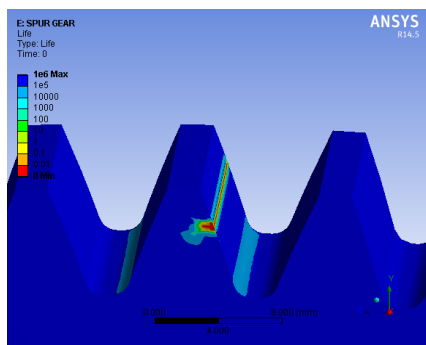


Figure 5: Life in number of cycles by Stress-life approach

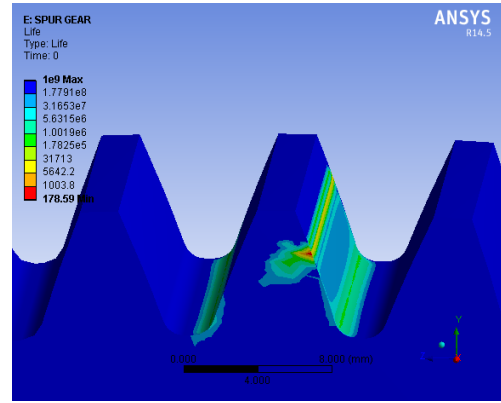


Figure 6: Life in number of cycles by strain-life approach

### Results and discussion

The results of the analysis carried out to study the influence of module on the design aspects of the spur gears are discussed in the following section. Fig. 7 shows the influence of module on the stress concentration factors at the root of the gear teeth. It can be observed that stress concentration factors decrease with increase in module, which can be attributed to increase in fillet radius at the root of the teeth, with increase in module.

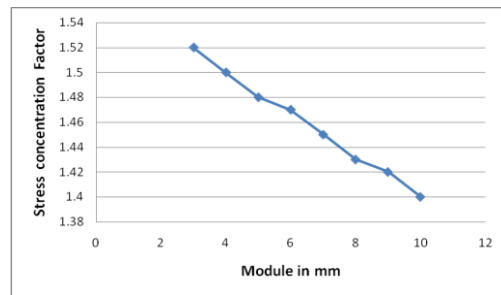


Figure 7: Module versus Stress concentration factor

Figure 8 shows the variation of bending stress at the root of the gear teeth, with module. The bending stress at the teeth root can be observed to decrease with increase in module. This can be attributed to increase in thickness of tooth at the root and decrease in stress concentration factor, with increase in module of the gear. The stress values obtained both analytically and numerically are found to be in close tolerance with each other. It can also be observed that the stress values become asymptotic at higher values of module.

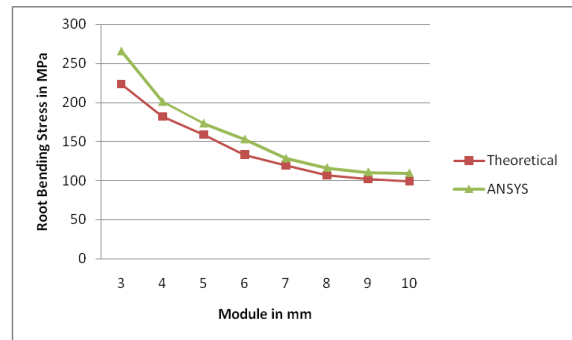


Figure 8: Module versus Root bending stress

Figure 9 indicates the stress based fatigue life of gears of different modules, determined analytically as well as numerically. It can be observed that fatigue life progressively increases with increases with module and beyond 5mm, the life become infinite for the operating conditions considered for the present study. Stress–life estimation is somewhat conservative since it doesn't take into account, the ability of the material to withstand localized plastic deformations.

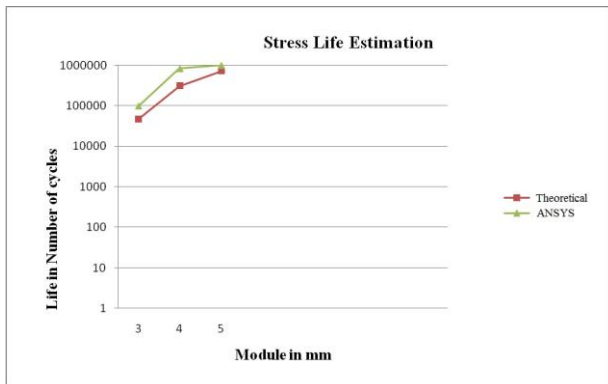


Figure 9: Module versus Life in number of cycles

Figure 10 indicates the strain based fatigue life of gears of different modules, determined analytically as well as numerically. The fatigue life increases with module and become asymptotic at higher values of module. Since the strain based approach recognizes the ability of the material to undergo localised plastic deformation without complete failure of the structure, the life estimation is more pragmatic than the stress based approach.

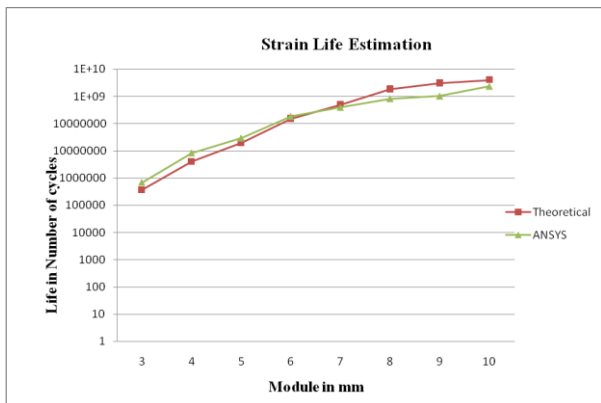


Figure 10: Module versus Life in number of cycles

Figure 11 shows the variation of gear mass with change in module. Moderate decrease in mass can be observed with increase in module.

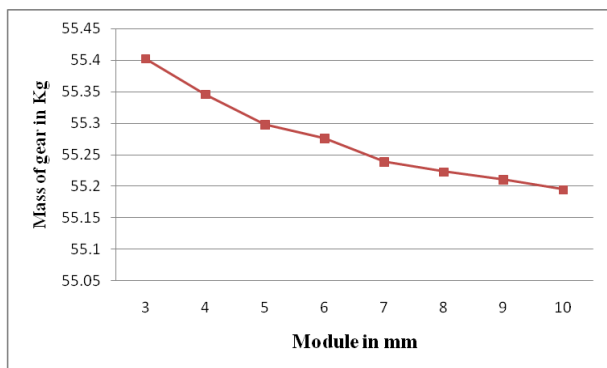


Figure 11: Module versus Mass of gear

### Conclusion

Present work is a small attempt to study the influence of module on design aspects of spur gears. Stress analysis and fatigue life estimation has been carried out for typical operating conditions, by varying the gear module. Results of the investigation indicate reduction in mass, stress concentration factor and stresses at the root of the teeth, with increase in gear module. Fatigue lives of the gears are also found to increase with gear module. The result obtained from analytical and numerical approaches are found to be consistent with each other.

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